



# **Calculation of Catastrophe Losses ceded to Reinsurance Contracts and Retained by Insurers**

XTREME LOSS SOLUTIONS White Paper

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## **Abstract**

This document reviews the reinsurance calculation formulas implemented in the XLS software system. The objective is to increase the transparency of the XLS software and to provide Users and Auditors with a reference to the formulas applied in reinsurance calculations.

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**TABLE OF CONTENTS**

<u>TABLE OF CONTENTS.....</u>	<u>2</u>
<u>AUDIENCE.....</u>	<u>3</u>
<u>EXECUTIVE SUMMARY.....</u>	<u>3</u>
<u>BACKGROUND &amp; DEFINITIONS.....</u>	<u>3</u>
<u>Quota Share Reinsurance Contracts.....</u>	<u>5</u>
<u>Key features</u>	<u>5</u>
<u>Formulas</u>	<u>5</u>
<u>Surplus Share Reinsurance Contracts.....</u>	<u>7</u>
<u>Key features</u>	<u>7</u>
<u>Formulas</u>	<u>7</u>
<u>Single Risk Excess of Loss Reinsurance Contracts.....</u>	<u>10</u>
<u>Key features</u>	<u>10</u>
<u>Formulas</u>	<u>10</u>
<u>Catastrophe Excess of Loss Reinsurance Contracts.....</u>	<u>14</u>
<u>Key features</u>	<u>14</u>
<u>Formulas</u>	<u>14</u>
<u>REINSURANCE FORMULAE FOR GROUPED EXPOSURE INFORMATION.....</u>	<u>17</u>
<u>Quota Share Reinsurance Contracts.....</u>	<u>18</u>
<u>Surplus Share Reinsurance Contracts.....</u>	<u>20</u>
<u>Single Risk Excess of Loss Reinsurance Contracts.....</u>	<u>22</u>
<u>Catastrophe Excess of Loss Reinsurance Contracts.....</u>	<u>22</u>



## **AUDIENCE**

This draft White Paper serves as a reference to XLS software Users and Auditors on the workings of reinsurance formulas embedded in the software.

## **EXECUTIVE SUMMARY**

The key features and reinsurance formulas of Quota Share, Surplus Share, Risk Excess of Loss and Catastrophe Excess of Loss reinsurance contracts are discussed in this document. In general, a formula for the loss ceded to the reinsurance contract is first derived followed by the formula for the retained loss. Reinsurance formulas are developed by first considering the basic features of each contract; and then expanded to include all other key features.

In the XLS software the user has the option to simulate catastrophe losses on grouped exposure information. Grouped exposure information is defined as summing ceded and retained sums insured per business segment per postal code. The ceded and retained losses are calculated by multiplying the ceded and retained sums insured with the simulated damage ratio. Reinsurance formulae for grouped exposure information are covered in the last section of this document.

## **BACKGROUND & DEFINITIONS**

The current version of the XLS software contains the following 3 types of Reinsurance contracts on the “Reinsurance Contracts” design screen:

- Quota Share
- Surplus Share
- Catastrophe Excess of Loss

Risk Excess of Loss contracts will be included in the next version of the software. However the key features and formulas of Risk Excess of loss contracts are included in this paper.

Quota Share and Surplus Share Reinsurance contracts are defined as proportional reinsurance contracts. It means a premium on one risk is ceded / retained in the same proportions as the risk is ceded / retained. The term Ceded relates to amounts / proportions of risk / claims to the reinsurance contract; and the term Retained relates to amounts / proportions of risk / claims to the insurer.

Risk Excess of Loss and Catastrophe Excess of Loss contracts are defined as non-proportional reinsurance contracts. Hence premiums on one risk is not ceded / retained in the same proportions as the risk is ceded / retained. Non-proportional contracts provide reinsurance cover above a certain amount, defined as the deductible, up to a certain amount, defined as the limit. Catastrophe Excess of Loss reinsurance does not provide protection to insurers on a risk by risk basis as with proportional contracts and Risk Excess of Loss contracts. A Catastrophe excess of loss contract protects the insurer against an accumulation of losses from a catastrophe event.

An insurer can purchase multiple reinsurance contracts, therefore the order of reinsurance contract calculations must be defined. The term used to specify the order of reinsurance calculations is “inurances”. Proportional contract calculations are done before non-proportional contract calculations, hence proportional contracts inures to the benefit of non-proportional contracts.



The following definitions and notations are used in the key features and formulas of reinsurance contracts below:

- Gross Sum Insured (GSI). The total gross insurable value of a risk.
- Gross Estimated Maximum Loss (GEML). The (GEML) of a risk is less than its (GSI). GEML's are normally specified for large commercial properties and relates to the maximum gross fire loss the property may suffer.
- Damage Ratio (DR). The damage ratio to a risk is simulated from a damage curve. Damage curves are discussed in the methodology papers of each natural peril.
- Gross per risk Catastrophe Loss (GL) = (DR) x (GSI). It is the amount of damage to one risk as a result of a catastrophe event. Note that damage ratios are applied to Gross Sums Insured (GSI) and not Gross Estimated Maximum Loss (GEML) values.
- Gross Catastrophe per business segment and postal code (GLBSPC). It is the total of per risk catastrophe losses per business segment and postal code.
- Ceded Loss (CL). The amount of the gross per risk Catastrophe Loss (GL) to a risk ceded to the reinsurance contract.
- Retained Loss (RL) = (GL) – (CL). The amount of the gross per risk Catastrophe Loss (GL) to a risk retained by the insurer after a reinsurance contract is applied.
- Total gross Catastrophe Loss (TGL). It is the sum of all Gross per risk Catastrophe Losses from risks covered by a Quota Share or Surplus Share contract.
- Retained Estimated Maximum Loss (REML). It is the amount of the (GEML) retained on a proportional reinsurance contract.
- Total retained per event catastrophe losses after all other reinsurance (TRPECLAOR).
- Total annual ceded catastrophe losses to a Catastrophe Excess of Loss layer (TACCLL)
- Ceded Loss per Event (CLPE). It is the amount of the per event catastrophe loss after proportional reinsurance ceded to the Catastrophe Excess of Loss layer.
- Retained Loss per Event (RLPE). It is the amount of the per event catastrophe loss after proportional reinsurance retained by the insurer.  
Retained Loss per Event (RLPE) = Total retained per event catastrophe losses after proportional reinsurance (TRPECLAPR) - Ceded Loss per Event (CLPE)
- Loss to Catastrophe Excess of Loss layer (LCEOLL). It is the loss to the catastrophe excess of loss layer before the maximum recovery against the layer is applied.
- Large Individual Loss after Proportional Reinsurance (LILAPR). It is a Large Individual Loss amount suffered by the insurer after all proportional reinsurances are applied as a result from a catastrophe event.
- Total annual ceded large individual losses to a Single Risk Excess of Loss layer (TACLSRL).
- Loss to Single Risk Excess of Loss layer (LSREOLL). It is the loss to the Single Risk Excess of Loss layer before the maximum recovery against the layer is applied.
- Ceded sum insured (CSI). It is the amount of the gross sum insured of a risk ceded to the Quota Share or Surplus Share contract.
- Total Ceded sum insured (TCSI). It is the sum of ceded sums insured of all risks covered by Quota Share or Surplus Share contracts

A business segment is a unique combination / permutation of the user descriptions of class of business, type of risk and source of business.

The following sections will define the key features of the various reinsurance contracts and its affect on retained / ceded calculations.



## **Quota Share Reinsurance Contracts**

### **Key features**

The key features and formula symbols of a Quota Share contract are:

- Retention percentage (RP)
- Cession percentage (CP) = 1 – (RP)
- Per risk capacity (PRC)
- Event limit (EL)

### **Formulas**

Each risk covered by a Quota Share contract will be retained and ceded in the proportions of (RP) and (CP). Therefore a Gross per risk Catastrophe Loss (GL) will be ceded and retained as:

$$\begin{aligned}\text{Ceded Loss (CL)} &= (\text{CP}) \times (\text{GL}) \\ \text{Retained Loss (RL)} &= (\text{RP}) \times (\text{GL})\end{aligned}$$

A Quota Share contract will also have a maximum Gross Estimated Maximum Loss (GEML) amount covered by the contract. This is defined as the “Per risk capacity” (PRC). If the insurer places a risk with an (GEML) greater than the (PRC) on the Quota Share and the risk produces a catastrophe claim then the Ceded Loss (CL) will be:

$$\text{Ceded Loss (CL)} = (\text{CP}) \times (\text{PRC}) / (\text{GEML}) \times (\text{GL})$$

Hence the Ceded Loss reduces by a multiplication factor of (PRC) / (GEML). Therefore the generalized Ceded Loss (CL) formula becomes:

If (GEML) > (PRC) then

$$\text{Ceded Loss (CL)} = (\text{CP}) \times (\text{GL}) \times (\text{PRC}) / (\text{GEML})$$

Else

$$\text{Ceded Loss (CL)} = (\text{CP}) \times (\text{GL})$$

As shown in the “Background & Definitions” section of this paper the Retained Loss simply becomes:

$$\text{Retained Loss (RL)} = (\text{GL}) - (\text{CL})$$

An Event limit will also be specified for a Quota Share contract. The Event limit (EL) is the maximum total gross catastrophe event loss (TGL) covered by the Quota Share contract. Hence if the total gross catastrophe loss from an event (TGL) exceeds the Event limit (EL) then no more per risk losses can be ceded to the Quota Share contract. Therefore:

If (TGL) > (EL) then Ceded Loss (CL) = 0



In the event of a large total gross catastrophe event loss (TGL) there is a point when the running total of (TGL) is just below the Event limit (EL). The next Gross per risk Catastrophe Loss (GL) pushes the running total of (TGL) above the Event limit (EL). The Ceded Loss (CL) for the risk that pushes the running total of (TGL) above the Event limit (EL) is calculated as:

$$\text{Ceded Loss (CL)} = (\text{CP}) \times ((\text{EL}) - ((\text{TGL}) - (\text{GL})))$$

Taking all of the key features of a Quota Share contract into account the generalized Ceded Loss (CL) formula for individual risk by risk information comes to:

If  $(\text{TGL}) > (\text{EL})$  then

If  $((\text{TGL}) - (\text{GL})) > (\text{EL})$  then

$$\text{Ceded Loss (CL)} = 0$$

Else

If  $(\text{GEML}) > (\text{PRC})$  then

$$\text{Ceded Loss (CL)} = (\text{CP}) \times (\text{PRC}) / (\text{GEML}) \times ((\text{EL}) - ((\text{TGL}) - (\text{GL})))$$

Else

$$\text{Ceded Loss (CL)} = (\text{CP}) \times ((\text{EL}) - ((\text{TGL}) - (\text{GL})))$$

Else

If  $(\text{GEML}) > (\text{PRC})$  then

$$\text{Ceded Loss (CL)} = (\text{CP}) \times (\text{PRC}) / (\text{GEML}) \times (\text{GL})$$

Else

$$\text{Ceded Loss (CL)} = (\text{CP}) \times (\text{GL})$$

The generalized Retained Loss (RL) simply becomes:

$$\text{Retained Loss (RL)} = (\text{GL}) - (\text{CL})$$

A Quota Share contract may "Inure" to the benefit of another proportional contract. The Retained Loss from the formula above will be incoming Retained Loss for the other proportional contract. Using the formulas and notation above the incoming Retained Loss becomes the "Gross per risk Catastrophe Loss (GL)" for the other proportional contract. A proportion of the (GEML) has been ceded to the Quota Share, therefore the other proportional contract would have a lower incoming retained estimated maximum loss (REML). The Retained Estimated Maximum Loss (REML) for the Quota Share contract is calculated as:

$$(*) \text{ Retained Estimated Maximum Loss (REML)} = ((\text{Retained Loss (RL)} / \text{Gross Loss (GL)}) \times \text{Gross Estimated Maximum Loss (GEML)})$$



The Retained Estimated Maximum Loss (REML) from the formula above will be incoming Retained Estimated Maximum Loss for the other proportional contract. Using the formulas and notation above the incoming Retained Estimated Maximum Loss becomes the “Gross Estimated Maximum Loss (GEML)” for the other proportional contract, except for the formula (\*) above. For the formula (\*) above (GL) remains the Gross per risk Catastrophe Loss (it does not become the incoming retained loss for the contract) and (GEML) remains the Gross Estimated Maximum Loss (it does not become the Retained Estimated Maximum Loss for the contract).

## **Surplus Share Reinsurance Contracts**

### **Key features**

The key features and formula symbols of a Surplus Share contract are:

- Retention amount (RA)
- Number of Surplus Lines (NSL). Each Surplus line provides per risk reinsurance cover equal to the Retention Amount (RA). By definition the maximum “Per Risk Capacity” placed on the Surplus Share contract is  $(1 + \text{Number of Surplus Lines}) \times \text{Retention amount (RA)}$ .
- Event limit (EL)

### **Formulas**

The proportion of the risk retained and ceded under a Surplus Share Contract depends on the retention amount (RA) and the number of surplus lines (NSL) in relation to the Gross Estimated Maximum Loss (GEML). Each risk covered by a Surplus Share contract will be ceded in the proportion:

If Gross Estimated Maximum Loss (GEML) < Retention Amount (RA) Then

Ceded proportion of risk = 0

Else

Ceded proportion of risk =  $(\text{Gross Estimated Maximum Loss (GEML)} - \text{Retention Amount (RA)}) / \text{Gross Estimated Maximum Loss (GEML)}$

Each risk covered by a Surplus Share contract will be retained in the proportion:

Retained proportion of risk =  $1 - \text{Ceded proportion of risk}$

Therefore a Gross per risk Catastrophe Loss (GL) will be ceded and retained as:

If Gross Estimated Maximum Loss (GEML) < Retention Amount (RA) Then

Ceded Loss (CL) = 0

Else

Ceded Loss (CL) =  $(\text{Gross Estimated Maximum Loss (GEML)} - \text{Retention Amount (RA)}) / \text{Gross Estimated Maximum Loss (GEML)} \times \text{Gross per risk Catastrophe Loss (GL)}$



The Retained Loss (RL) is calculated as:

$$\text{Retained Loss (RL)} = \text{Gross per risk Catastrophe Loss (GL)} - \text{Ceded Loss (CL)}$$

By definition the maximum GEML amount covered by the Surplus Share contract is ( 1 + Number of Surplus Lines ) x Retention amount (RA). If the insurer places a risk with an (GEML) greater than the

maximum GEML amount on the Surplus Share contract and the risk produces a catastrophe claim then the Ceded Loss (CL) will be:

If Gross Estimated Maximum Loss (GEML) < Retention Amount (RA) Then

$$\text{Ceded Loss (CL)} = 0$$

Else

If Gross Estimated Maximum Loss (GEML) > Retention Amount (RA) x ( 1 + Number of Surplus Lines (NSL) ) Then

$$\text{Ceded Loss (CL)} = \text{Number of Surplus Lines (NSL)} \times \text{Retention Amount (RA)} / \text{Gross Estimated Maximum Loss (GEML)} \times \text{Gross per risk Catastrophe Loss (GL)}$$

Else

$$\text{Ceded Loss (CL)} = ( \text{Gross Estimated Maximum Loss (GEML)} - \text{Retention Amount (RA)} ) / \text{Gross Estimated Maximum Loss (GEML)} \times \text{Gross per risk Catastrophe Loss (GL)}$$

The Retained Loss (RL) is calculated as:

$$\text{Retained Loss (RL)} = \text{Gross per risk Catastrophe Loss (GL)} - \text{Ceded Loss (CL)}$$

An Event limit will also be specified for a Surplus Share contract. The Event limit (EL) is the maximum total gross catastrophe event loss covered by the Surplus Share contract (TGL). Hence if the total gross catastrophe loss from an event (TGL) exceeds the Event limit (EL) then no more per risk losses can be ceded to the Surplus Share contract. Therefore:

If (TGL) > (EL) then Ceded Loss (CL) = 0

In the event of a large total gross catastrophe event loss (TGL) there is a point when the running total of (TGL) is just below the Event limit (EL). The next Gross per risk Catastrophe Loss (GL) pushes the running total of (TGL) above the Event limit (EL). The Ceded Loss (CL) for the risk that pushes the running total of (TGL) above the Event limit (EL) is calculated as:

$$\text{Ceded Loss (CL)} = ( \text{Gross Estimated Maximum Loss (GEML)} - \text{Retention Amount (RA)} ) / \text{Gross Estimated Maximum Loss (GEML)} \times ( \text{EL} - ( \text{TGL} - \text{GL} ) )$$



Taking all of the key features of a Surplus Share contract into account the generalized Ceded Loss (CL) formula for individual risk by risk information comes to:

If  $(TGL) > (EL)$  then

If  $((TGL) - (GL)) > (EL)$  then

Ceded Loss (CL) = 0

Else

If Gross Estimated Maximum Loss (GEML) < Retention Amount (RA) Then

Ceded Loss (CL) = 0

Else

If Gross Estimated Maximum Loss (GEML) > Retention Amount (RA) x ( 1 + Number of Surplus Lines (NSL) ) Then

Ceded Loss (CL) = Number of Surplus Lines (NSL) x Retention Amount (RA) / Gross Estimated Maximum Loss (GEML) x ( (EL) - ((TGL) - (GL)) )

Else

Ceded Loss (CL) = ( Gross Estimated Maximum Loss (GEML) - Retention Amount (RA) ) / Gross Estimated Maximum Loss (GEML) x ( (EL) - ((TGL) - (GL)) )

Else

If Gross Estimated Maximum Loss (GEML) < Retention Amount (RA) Then

Ceded Loss (CL) = 0

Else

If Gross Estimated Maximum Loss (GEML) > Retention Amount (RA) x ( 1 + Number of Surplus Lines (NSL) ) Then

Ceded Loss (CL) = Number of Surplus Lines (NSL) x Retention Amount (RA) / Gross Estimated Maximum Loss (GEML) x (GL)

Else

Ceded Loss (CL) = ( Gross Estimated Maximum Loss (GEML) - Retention Amount (RA) ) / Gross Estimated Maximum Loss (GEML) x (GL)



The generalized Retained Loss (RL) simply becomes:

$$\text{Retained Loss (RL)} = (\text{GL}) - (\text{CL})$$

A Surplus Share contract may “Inure” to the benefit of another proportional contract. The Retained Loss from the formula above will be incoming Retained Loss for the other proportional contract. Using the formulas and notation above the incoming Retained Loss becomes the “Gross per risk Catastrophe Loss (GL)” for the other proportional contract. A proportion of the (GEML) has been ceded to the Surplus Share contract, therefore the other proportional contract would have a lower incoming retained (REML). The Retained Estimated Maximum Loss (REML) for the Surplus Share contract is calculated as:

$$(*) \text{ Retained Estimated Maximum Loss (REML)} = ( \text{Retained Loss (RL)} / \text{Gross Loss (GL)} ) \times \text{Gross Estimated Maximum Loss (GEML)}$$

The Retained Estimated Maximum Loss (REML) from the formula above will be incoming Retained Estimated Maximum Loss for the other proportional contract. Using the formulas and notation above the incoming Retained Estimated Maximum Loss becomes the “Gross Estimated Maximum Loss (GEML)” for the other proportional contract, except for the formula (\*) above. For the formula (\*) above (GL) remains the Gross per risk Catastrophe Loss (it does not become the incoming retained loss for the contract) and (GEML) remains the Gross Estimated Maximum Loss (it does not become the Retained Estimated Maximum Loss for the contract).

## **Single Risk Excess of Loss Reinsurance Contracts**

This type of contract will be included in the new version of XLS software. Quota Share and Surplus Share contracts inure to the benefit of Single Risk Excess of Loss contracts. Single Risk Excess of Loss contracts in turn inure to the benefit of Catastrophe Excess of Loss Contracts.

### **Key features**

The key features and formula symbols of a Single Risk Excess of Loss contract are:

- Deductible amount (DA). The retained large individual loss after all proportional contracts must exceed the Deductible Amount (DA) before reinsurance cover from the Single Risk Excess of Loss contract starts. Single Risk Excess of Loss contracts are sold as layers stacking on top of each other.
- Limit amount (LA). The Limit Amount (LA) is the maximum amount the insurer can recover from a Single Risk Excess of Loss layer per large individual loss after all proportional reinsurance. A catastrophe event can result in a number of large individual losses.
- Percentage reinsured (PR). It is the percentage of a Risk Excess of Loss layer reinsured.
- Number of reinstatements (NR). It is the total number of times over an annual period the limit of a Single Risk Excess of Loss layer can be reinstated. The maximum annual recovery an insurer can make from a Single Risk Excess of Loss layer is  $( 1 + \text{Number of reinstatements (NR)} ) \times \text{Limit amount (LA)}$ . The reinsurance cover on a layer is reinstated when the total recoveries from large individual losses on the layer exceed the Limit amount (LA) of the layer.



## **Formulas**

Single Risk Excess of Loss reinsurance protects the insurer's net account against large individual losses after proportional reinsurance. To apply Single Risk Excess of Loss reinsurance cover the Large Individual Loss after proportional reinsurance (LILAPR) is required.

If the Large Individual Loss after proportional reinsurance (LILAPR) is less than the Deductible amount (DA) on a layer then no reinsurance recovery can be made. Therefore:

If  $(LILAPR) < \text{Deductible amount (DA)}$  then  $\text{Ceded Loss (CL)} = 0$

$\text{Retained Loss (RL)} = \text{Large Individual Loss after proportional reinsurance (LILAPR)}$

If the Large Individual Loss after proportional reinsurance (LILAPR) is greater than the Deductible amount (DA) on a layer then reinsurance recovery can be made. In this case two outcomes are possible:

- The Large Individual Loss after proportional reinsurance (LILAPR) is a large amount and exceeds the Deductible amount (DA) plus the Limit amount (LA) on the layer. For this outcome the insurer can only recover a maximum amount per large individual loss equal to the Limit amount (LA). Therefore:

If  $\text{Large Individual Loss after proportional reinsurance (LILAPR)} > \text{Deductible amount (DA)} + \text{Limit amount (LA)}$  then  $\text{Ceded Loss (CL)} = \text{Limit amount (LA)}$

The retained loss (RL) would be:

$\text{Retained Loss (RL)} = \text{Large Individual Loss after proportional reinsurance (LILAPR)} - \text{Ceded loss (CL)}$

- The Large Individual Loss after proportional reinsurance (LILAPR) exceeds the Deductible amount (DA) but is less than the Deductible amount (DA) plus the Limit amount (LA) on the layer. For this outcome the insurer recovers a proportion of the Limit amount (LA) on the layer. Therefore:

If  $\text{Deductible amount (DA)} < (LILAPR) < \text{Deductible amount (DA)} + \text{Limit amount (LA)}$  then  $\text{Ceded Loss (CL)} = \text{Large Individual Loss after proportional reinsurance (LILAPR)} - \text{Deductible amount (DA)}$

The retained loss (RL) would be:

$\text{Retained Loss (RL)} = \text{Large Individual Loss after proportional reinsurance (LILAPR)} - \text{Ceded loss (CL)}$

Only a certain percentage of the layer maybe reinsured (PR). If this is the case all Ceded Losses (CL) must be multiplied by the percentage reinsured (PR).

If the running total of annual ceded losses to a Single Risk Excess of Loss layer (TACLSRL) exceeds the  $(\text{Number of reinstatements} + 1) \times \text{Limit amount (LA)}$  then no more recoveries can be made against the Single Risk Excess of Loss layer.



First we calculate the Loss to the Single Risk Excess of Loss Layer (LSREOLL) on each individual risk affected by the catastrophe event:

If Large Individual Loss after proportional reinsurance (LILAPR) < Deductible amount (DA) then

$$\text{Loss to the Single Risk Excess of Loss Layer (LSREOLL)} = 0$$

Else

If Large Individual Loss after proportional reinsurance (LILAPR) > Deductible amount (DA) + Limit amount (LA) then

$$\text{Loss to the Single Risk Excess of Loss Layer (LSREOLL)} = \text{Limit amount (LA)}$$

Else

$$\text{Loss to the Single Risk Excess of Loss Layer (LSREOLL)} = \text{Large Individual Loss after proportional reinsurance (LILAPR)} - \text{Deductible amount (DA)}$$

If total annual ceded large individual losses to a Single Risk Excess of Loss layer (TACLSRL) > (Number of reinstatements +1) x Limit amount (LA) then

$$\text{Ceded Loss (CL)} = (\text{Number of reinstatements} + 1) \times \text{Limit amount (LA)} - (\text{Total annual ceded large individual losses to a Single Risk Excess of Loss layer (TACLSRL)} - \text{Loss to the Single Risk Excess of Loss Layer (LSREOLL)})$$

$$\text{Retained Loss (RL)} = \text{Large Individual Loss after proportional reinsurance (LILAPR)} - \text{Ceded Loss (CL)}$$

If the reinsurance cover on the layer has been exhausted before the current large individual loss then the Ceded Loss (CL) is 0. Hence:

$$\text{If Total annual ceded large individual losses to a Single Risk Excess of Loss layer (TACLSRL)} = (\text{Number of reinstatements} + 1) \times \text{Limit amount (LA)} \text{ then Ceded Loss (CL)} = 0$$

$$\text{Retained Loss (CL)} = \text{Large Individual Loss after proportional reinsurance (LILAPR)} - \text{Ceded Loss (CL)}$$



Taking all of the key features of a Single Risk Excess of loss contract into account the generalized Ceded Loss (CL) formula for individual risk by risk information comes to:

If Large Individual Loss after proportional reinsurance (LILAPR) < Deductible amount (DA) then

$$\text{Loss to the Single Risk Excess of Loss Layer (LSREOLL)} = 0$$

Else

If Large Individual Loss after proportional reinsurance (LILAPR) > Deductible amount (DA) + Limit amount (LA) then

$$\text{Loss to the Single Risk Excess of Loss Layer (LSREOLL)} = \text{Limit amount (LA)}$$

Else

$$\text{Loss to the Single Risk Excess of Loss Layer (LSREOLL)} = \text{Large Individual Loss after proportional reinsurance (LILAPR)} - \text{Deductible amount (DA)}$$

The maximum recovery against the Single Risk Excess of Loss layer must be applied after the loss to the layer is calculated for each individual risk affected by the catastrophe event. Please note that the total annual ceded large individual losses to a Single Risk Excess of Loss layer (TACLSRL) in the formulas below refer to the variable value before the ceded loss on the current large individual loss is added.

If Total annual ceded large individual losses to a Single Risk Excess of Loss layer (TACLSRL) = (Number of reinstatements +1) x Limit amount (LA) then

$$\text{Ceded Loss (CL)} = 0$$

Else

If Total annual ceded large individual losses to a Single Risk Excess of Loss layer (TACLSRL) + Loss to the Single Risk Excess of Loss Layer (LSREOLL) > (Number of reinstatements +1) x Limit amount (LA) then

$$\text{Ceded Loss (CL)} = ( \text{Number of reinstatements} +1 ) \times \text{Limit amount (LA)} - \text{Total annual ceded large individual losses to a Single Risk Excess of Loss layer (TACLSRL)} ) \times \text{Percentage Reinsured (PR)}$$

Else

$$\text{Ceded Loss (CL)} = \text{Loss to the Single Risk Excess of Loss Layer (LSREOLL)} \times \text{Percentage Reinsured (PR)}$$

The new total of annual ceded large individual losses to a Single Risk Excess of Loss layer (TACLSRL) after this large individual loss is then calculated as:



Total annual ceded large individual losses to a Single Risk Excess of Loss layer (TACLSRL) = Total annual ceded large individual losses to a Single Risk Excess of Loss layer (TACLSRL) + Ceded Loss (CL) / Percentage Reinsured (PR)

Retained Loss (RL) = Large Individual Loss after proportional reinsurance (LILAPR) - Ceded Loss (CL)

## **Catastrophe Excess of Loss Reinsurance Contracts**

### **Key features**

The key features and formula symbols of a Catastrophe Excess of Loss contract are:

- Deductible amount (DA). The total of retained catastrophe losses per event after all proportional contracts must exceed the Deductible Amount (DA) before reinsurance cover from the Catastrophe Excess of Loss contract starts. Catastrophe Excess of Loss contracts are sold as layers stacking on top of each other.
- Limit amount (LA). The Limit Amount (LA) is the maximum amount the insurer can recover from a layer per catastrophe event from the Catastrophe Excess of Loss contract.
- Percentage reinsured (PR). It is the percentage of a Catastrophe Excess of Loss layer reinsured.
- Number of reinstatements (NR). It is the total number of times over an annual period the limit of a Catastrophe Excess of Loss layer can be reinstated. The maximum annual recovery an insurer can make from a Catastrophe Excess of Loss layer is  $(1 + \text{Number of reinstatements (NR)}) \times \text{Limit amount (LA)}$ . The reinsurance cover on a layer is reinstated when the total recoveries from catastrophe events on the layer exceed the Limit amount (LA) of the layer.

### **Formulas**

Catastrophe excess of loss reinsurance protects the insurer's net account after all other reinsurance. To apply Catastrophe Excess of Loss reinsurance cover the Total Retained per event catastrophe loss after all other reinsurance (TRPECLAOR) is required. All other reinsurance refers to Quota Share, Surplus Share and Single Risk Excess of Loss reinsurance contracts.

If the Total Retained per event catastrophe loss after all other reinsurance (TRPECLAOR) is less than the Deductible amount (DA) on a layer then no reinsurance recovery can be made. Therefore:

If  $(\text{TRPECLAOR}) < \text{Deductible amount (DA)}$  then Ceded Loss per Event (CLPE) = 0

Retained Loss per event (RLPE) = Total Retained per event catastrophe loss after all other reinsurance (TRPECLAOR)

If the Total Retained per event catastrophe loss after all other reinsurance (TRPECLAOR) is greater than the Deductible amount (DA) on a layer then reinsurance recovery can be made. In this case two outcomes are possible:

- The Total Retained per event catastrophe loss after all other reinsurance (TRPECLAOR) is a large amount and exceeds the Deductible amount (DA) plus the Limit amount (LA) on the layer. For this outcome the insurer can only recover a maximum amount equal to the Limit amount (LA). Therefore:



If  $(TRPECLAOR) > \text{Deductible amount (DA)} + \text{Limit amount (LA)}$  then Ceded Loss per event (CLPE) = Limit amount (LA)

The retained loss per event (RLPE) would be:

Retained Loss per event (RLPE) = Total Retained per event catastrophe loss after all other reinsurance (TRPECLAOR) – Ceded loss per event (CLPE)

- The Total Retained per event catastrophe loss after all other reinsurance (TRPECLAOR) exceeds the Deductible amount (DA) but is less than the Deductible amount (DA) plus the Limit amount (LA) on the layer. For this outcome the insurer recovers a proportion of the Limit amount (LA) on the layer. Therefore:

If  $\text{Deductible amount (DA)} < (TRPECLAOR) < \text{Deductible amount (DA)} + \text{Limit amount (LA)}$  then Ceded Loss per event (CLPE) = Total Retained per event catastrophe loss after all other reinsurance (TRPECLAOR) - Deductible amount (DA)

The retained loss per event (RLPE) would be:

Retained Loss per event (RLPE) = Total Retained per event catastrophe loss after all other reinsurance (TRPECLAOR) – Ceded loss (CLPE)

Only a certain percentage of the layer maybe reinsured (PR). If this is the case all Ceded Losses per event (CLPE) must be multiplied by the percentage reinsured (PR).

If the running total of annual ceded catastrophe losses to a Catastrophe Excess of Loss layer (TACCLL) exceeds the  $(\text{Number of reinstatements} + 1) \times \text{Limit amount (LA)}$  then no more recoveries can be made against the Catastrophe Excess of Loss layer.

First we calculate the Loss to the Catastrophe Excess of Loss Layer (LCEOLL):

If  $(TRPECLAOR) < \text{Deductible amount (DA)}$  then

Loss to the Catastrophe Excess of Loss Layer (LCEOLL) = 0

Else

If  $(TRPECLAOR) > \text{Deductible amount (DA)} + \text{Limit amount (LA)}$  then

Loss to the Catastrophe Excess of Loss Layer (LCEOLL) = Limit amount (LA)

Else

Loss to the Catastrophe Excess of Loss Layer (LCEOLL) =  $(TRPECLAOR) - \text{Deductible amount (DA)}$

If total of annual ceded catastrophe losses to a Catastrophe Excess of Loss layer (TACCLL)  $> (\text{Number of reinstatements} + 1) \times \text{Limit amount (LA)}$  then



Ceded Loss per event (CLPE) = (Number of reinstatements +1) x Limit amount (LA) – ( Total of annual ceded catastrophe losses to a Catastrophe Excess of Loss layer (TACCLL) - Loss to the Catastrophe Excess of Loss Layer (LCEOLL) )

Retained Loss per event (CLPE) = Total Retained per event catastrophe loss after all other reinsurance (TRPECLAOR) - Ceded Loss per event (CLPE)

If the reinsurance cover on the layer has been exhausted before the current catastrophe event then the Ceded Loss per event (CLPE) is 0. Hence:

If Total of annual ceded catastrophe losses to a Catastrophe Excess of Loss layer (TACCLL) = (Number of reinstatements +1) x Limit amount (LA) then Ceded Loss per event (CLPE) = 0

Retained Loss per event (RLPE) = Total Retained per event catastrophe loss after all other reinsurance (TRPECLAOR) - Ceded Loss per event (CLPE)

Taking all of the key features of a Catastrophe Excess of loss contract into account the generalized Ceded Loss per Event (CLPE) formula comes to:

If (TRPECLAOR) < Deductible amount (DA) then

Loss to the Catastrophe Excess of Loss Layer (LCEOLL) = 0

Else

If (TRPECLAOR) > Deductible amount (DA) + Limit amount (LA) then

Loss to the Catastrophe Excess of Loss Layer (LCEOLL) = Limit amount (LA)

Else

Loss to the Catastrophe Excess of Loss Layer (LCEOLL) = (TRPECLAOR) - Deductible amount (DA)

The maximum recovery against the Catastrophe Excess of Loss layer must be applied after the loss to the layer is calculated. Please note that the total of annual ceded catastrophe losses to a Catastrophe Excess of Loss layer (TACCLL) in the formulas below refer to the variable value before the ceded loss on the current event is added.



If Total of annual ceded catastrophe losses to a Catastrophe Excess of Loss layer (TACCLL) = (Number of reinstatements +1) x Limit amount (LA) then

$$\text{Ceded Loss per event (CLPE)} = 0$$

Else

If Total of annual ceded catastrophe losses to a Catastrophe Excess of Loss layer (TACCLL) + Loss to the Catastrophe Excess of Loss Layer (LCEOLL) > (Number of reinstatements +1) x Limit amount (LA) then

$$\text{Ceded Loss per event (CLPE)} = ( (\text{Number of reinstatements} + 1) \times \text{Limit amount (LA)} - \text{Total of annual ceded catastrophe losses to a Catastrophe Excess of Loss layer (TACCLL)} ) \times \text{Percentage Reinsured (PR)}$$

Else

$$\text{Ceded Loss per event (CLPE)} = \text{Loss to the Catastrophe Excess of Loss Layer (LCEOLL)} \times \text{Percentage Reinsured (PR)}$$

The new total of annual ceded catastrophe losses to a Catastrophe Excess of Loss layer (TACCLL) after this event is then calculated as:

$$\text{Total of annual ceded catastrophe losses to a Catastrophe Excess of Loss layer (TACCLL)} = \text{Total of annual ceded catastrophe losses to a Catastrophe Excess of Loss layer (TACCLL)} + \text{Ceded Loss per event (CLPE)} / \text{Percentage Reinsured (PR)}$$

$$\text{Retained Loss per event (RLPE)} = \text{Total retained per event catastrophe losses after all other reinsurance (TRPECLAOR)} - \text{Ceded Loss per event (CLPE)}$$

## **REINSURANCE FORMULAE FOR GROUPED EXPOSURE INFORMATION**

In order to use reinsurance contract reports in the XLS software, imported exposure information must be risk by risk data. Hence one line / record of exposure information presents one risk. The user can import grouped exposure information (i.e. total sums insured per business segment per postal code) but then reinsurance reports will be inaccurate.



## **Quota Share Reinsurance Contracts**

First, the ceded and retained sums insured per risk must be calculated and summed per business segment per postal code per reinsurance contract. Thus for a Quota Share contract:

If (GEML) > (PRC) then

$$\text{Ceded Sum Insured (CSI)} = (\text{CP}) \times (\text{PRC}) / (\text{GEML}) \times (\text{GSI})$$

Else

$$\text{Ceded Sum Insured (CSI)} = (\text{CP}) \times (\text{GSI})$$

The Retained Sum Insured (RSI) per risk is calculated as:

$$\text{Retained Sum Insured (RSI)} = \text{Gross Sum Insured (GSI)} - \text{Ceded Sum Insured (CSI)}$$

$$\text{Total Ceded Sum Insured (TCSI)} = \sum (\text{CSI})$$

$$\text{Total Retained Sum Insured (TRSI)} = \sum (\text{RSI})$$

Next, the ceded and retained Estimated Maximum Loss (EML) per risk must be calculated and summed per business segment per postal code per reinsurance contract. This information is required if the Quota Share inures to the benefit of another proportional contract. Thus for a Quota Share contract:

If (GEML) > (PRC) then

$$\text{Ceded Estimated Maximum Loss (CEML)} = (\text{CP}) \times (\text{PRC})$$

Else

$$\text{Ceded Estimated Maximum Loss (CEML)} = (\text{CP}) \times (\text{GEML})$$

The Retained Estimated Maximum Loss (REML) per risk is calculated as:

$$\text{Retained Estimated Maximum Loss (REML)} = \text{Gross Estimated Maximum Loss (GEML)} - \text{Ceded Estimated Maximum Loss (CEML)}$$

If the Quota Share contract comes after another proportional reinsurance contract then (GEML) and (GSI) in the formulas above equal the Retained Estimated Maximum Loss (REML) and Retained Sum Insured (RSI) of the other proportional contract.

A Damage Ratio (DR) will be simulated for each business segment at a postal code. The Gross Loss per business segment and postal codes (GLBSPC) is then calculated as:

$$\text{Gross Loss per business segment and postal codes (GLBSPC)} = \text{Damage ratio (DR) per business segment and postal code} \times (\text{Total Ceded Sum Insured per business segment and postal code (TCSI)} + \text{Total Retained Sum Insured per business segment and postal code (TRSI)})$$



Next the running total of the Total Gross Loss over business segments and postal codes (TGLBSPC) has to be compared with the Event Limit (EL) in order to calculate the Ceded Loss (CL) per business segment and postal code.

If  $(TGLBSPC) > (EL)$  then

If  $((TGLBSPC) - (GLBSPC)) > (EL)$  then

Ceded Loss (CL) = 0

Else

Ceded Loss (CL) = Total Ceded Sum Insured (TCSI) of business segment and postal code for the Quota Share contract / (Total Ceded Sum Insured per business segment and postal code (TCSI) for the Quota Share Contract + Total Retained Sum Insured per business segment and postal code (TRSI) )  
 $\times ((EL) - ((TGLBSPC) - (GLBSPC)))$

Else

Ceded Loss (CL) = Total Ceded Sum Insured (TCSI) of business segment and postal code x Damage ratio (DR) per business segment and postal code

The Retained Loss (RL) per business segment and postal code simply becomes:

Retained Loss (RL) =  $(GLBSPC) - (CL)$

A Quota Share contract may “Inure” to the benefit of another proportional contract. The Retained Loss from the formula above will be incoming Retained Loss for the other proportional contract. Using the formulas and notation above the incoming Retained Loss becomes the “Gross Loss per business segment and postal codes (GLBSPC)” for the other proportional contract.



## **Surplus Share Reinsurance Contracts**

First, the ceded and retained sums insured per risk must be calculated and summed per business segment per postal code per reinsurance contract. Thus for a Surplus Share contract:

If Gross Estimated Maximum Loss (GEML) < Retention Amount (RA) Then

$$\text{Ceded Sum Insured (CSI)} = 0$$

Else

If Gross Estimated Maximum Loss (GEML) > Retention Amount (RA) x ( 1 + Number of Surplus Lines (NSL) ) Then

$$\text{Ceded Sum Insured (CSI)} = \text{Number of Surplus Lines (NSL)} \times \text{Retention Amount (RA)} / \text{Gross Estimated Maximum Loss (GEML)} \times (\text{GSI})$$

Else

$$\text{Ceded Sum Insured (CSI)} = (\text{Gross Estimated Maximum Loss (GEML)} - \text{Retention Amount (RA)}) / \text{Gross Estimated Maximum Loss (GEML)} \times (\text{GSI})$$

The Retained Sum Insured (RSI) per risk is calculated as:

$$\text{Retained Sum Insured (RSI)} = \text{Gross Sum Insured (GSI)} - \text{Ceded Sum Insured (CSI)}$$

$$\text{Total Ceded Sum Insured (TCSI)} = \sum (\text{CSI})$$

$$\text{Total Retained Sum Insured (TRSI)} = \sum (\text{RSI})$$

Next, the ceded and retained Estimated Maximum Loss (EML) per risk must be calculated and summed per business segment per postal code per reinsurance contract. This information is required if the Surplus Share inures to the benefit of another proportional contract. Thus for a Surplus Share contract:

If Gross Estimated Maximum Loss (GEML) < Retention Amount (RA) Then

$$\text{Ceded Estimated Maximum Loss (CEML)} = 0$$

Else

If Gross Estimated Maximum Loss (GEML) > Retention Amount (RA) x ( 1 + Number of Surplus Lines (NSL) ) Then

$$\text{Ceded Estimated Maximum Loss (CEML)} = \text{Number of Surplus Lines (NSL)} \times \text{Retention Amount (RA)}$$

Else

$$\text{Ceded Estimated Maximum Loss (CEML)} = (\text{Gross Estimated Maximum Loss (GEML)} - \text{Retention Amount (RA)})$$



The Retained Estimated Maximum Loss (REML) per risk is calculated as:

Retained Estimated Maximum Loss (REML) = Gross Estimated Maximum Loss (GEML) - Ceded Estimated Maximum Loss (CEML)

If the Surplus Share contract comes after another proportional reinsurance contract then (GEML) and (GSI) in the formulas above equal the Retained Estimated Maximum Loss (REML) and Retained Sum Insured (RSI) of the other proportional contract.

A Damage Ratio (DR) will be simulated for each business segment at a postal code. The Gross Loss per business segment and postal codes (GLBSPC) is then calculated as:

Gross Loss per business segment and postal codes (GLBSPC) = Damage ratio (DR) per business segment and postal code x (Total Ceded Sum Insured per business segment and postal code (TCSI) + (Total Retained Sum Insured per business segment and postal code (TRSI) )

Next the running total of the Total Gross Loss over business segments and postal codes (TGLBSPC) has to be compared with the Event Limit (EL) in order to calculate the Ceded Loss (CL) per business segment and postal code.

If (TGLBSPC) > (EL) then

If ((TGLBSPC) – (GLBSPC) ) > (EL) then

Ceded Loss (CL) = 0

Else

Ceded Loss (CL) = Total Ceded Sum Insured (TCSI) of business segment and postal code for the Quota Share contract / (Total Ceded Sum Insured per business segment and postal code (TCSI) for the Quota Share Contract + Total Retained Sum Insured per business segment and postal code (TRSI) ) x ( (EL) – ((TGLBSPC) – (GLBSPC) ) )

Else

Ceded Loss (CL) = Total Ceded Sum Insured (TCSI) of business segment and postal code x Damage ratio (DR) per business segment and postal code

The Retained Loss (RL) per business segment and postal code simply becomes:

Retained Loss (RL) = (GLBSPC) – (CL)

A Surplus Share contract may “Inure” to the benefit of another proportional contract. The Retained Loss from the formula above will be incoming Retained Loss for the other proportional contract. Using the formulas and notation above the incoming Retained Loss becomes the “Gross Loss per business segment and postal codes (GLBSPC)” for the other proportional contract.



### **Single Risk Excess of Loss Reinsurance Contracts**

Ceded losses to Single Risk Excess of Loss Reinsurance Contracts cannot be calculated if exposure information summed by business segment and postal code. Losses to individual risks must be known to calculate ceded losses to Single Risk Excess of Loss Reinsurance Contracts.

In the XLS software Ceded losses to Single Risk Excess of Loss Reinsurance Contracts will be set equal to R 0, therefore Retained Losses before and after Single Risk Excess of Loss Reinsurance Contracts will be equal.

### **Catastrophe Excess of Loss Reinsurance Contracts**

Reinsurance Formulas under grouped exposure information is the same as risk by risk exposure information. The reason is Catastrophe Excess of Loss contracts apply to the total per catastrophe event retained loss after proportional reinsurance which is the same under grouped and risk by risk exposure information.